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Magnetic control of thermal convection in electrically non-conducting or low-conducting paramagnetic fluids

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Abstract

An inhomogeneous magnetic field exerts a magnetization force on all materials, including electrically conducting and non-conducting fluids. A recent experiment demonstrated that this force can enhance or suppress convection in a paramagnetic aqueous solution heated from either below or above. In this paper, to clarify the mechanism of the observed phenomena, we numerically simulated the effect of a vertical magnetic field gradient on thermal convection in paramagnetic fluids. These simulated results agree with experimental observations. Our study shows that the magnetic buoyancy force has potential applications for controlling convection and enhancing the heat transfer efficiency in either electrically non-conducting or low-conducting fluids.

1. Introduction

Many studies have been done on magnetically controlled convection in electrically conducting fluids [1]. When an electrically conducting fluid, such as a liquid metal or molten silicon, moves in a magnetic field, Lorentz forces dampen the fluid motion. In particular, the application of magnetic fields has been used in various industrial fields, for example, magnetic suppression of convection in molten semiconductor material and magnetic control of molten iron flows in the steel industry.

However, Lorentz forces do not apply to electrically non-conducting or low-conducting fluids, such as melts of inorganic oxides, organic solvents, and aqueous solutions with low conductivity. Except for the works related to ferromagnetic fluids, few studies have been made on the effects that magnetic fields may have on convection in either electrically non-conducting or low-conducting fluids [2–6]. In these studies, instead of Lorentz forces, magnetization forces, which act on every material, were used to control convection. In an inhomogeneous magnetic field, a unit volume of material experiences a magnetization force that can be expressed as [7]:

$$\mathbf{F}_{\mathrm{m}} = \frac{1}{2}\mu_0\chi\nabla\mathbf{H}^2 = \frac{1}{2}\mu_0\rho\chi_{\mathrm{g}}\nabla\mathbf{H}^2. \tag{1}$$

It is characteristic that $\mathbf{F}_{\mathrm{m}}$ depends only on the gradient of the magnetic field ($\nabla\mathbf{H}^2$) but not on the field direction. The magnetization force ($\mathbf{F}_{\mathrm{m}}$) is a body force, and is analogous to the gravitational force ($\mathbf{F}_{\mathrm{g}} = \rho\mathbf{g}$), where χ is similar to the density, and $\nabla\mathbf{H}^2$ exerts a body force similar to gravity. Most material can be classified as either paramagnetic (PM) or diamagnetic (DM). Magnetic fields create a large attractive force on the former material, and a weak repulsive force on the latter material.

Recently, the magnetic attractive force acting on oxygen gas, which is paramagnetic, has been found to affect air convection and to promote combustion through magnetoaerodynamics [2,3]. Furthermore, buoyancy can be changed by applying vertical magnetization forces in an aqueous solution [4–6]. Braithwaite et al. [4] have observed both strong enhancing and suppressing effects of an inhomogeneous magnetic field

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Nomenclature

C_p	specific heat at constant pressure
D	thermal diffusivity, $k/\rho C_p$
$\mathbf{F}_m$	magnetization force
$\mathbf{F}_L$	Lorentz force, $\mu[\mathbf{j} \times \mathbf{H}]$
$\mathbf{F}_g$	gravitational body force
$\mathbf{g}$	acceleration of gravity
h	cell height
$\mathbf{H}$	magnetic field intensity
H	magnitude of magnetic field intensity
$\mathbf{j}$	electrical current density
k	thermal conductivity
p	pressure
t	time
T	temperature
$\mathbf{v}$	velocity vector
w	cell width
x, z	spatial Cartesian coordinates
Greek symbols	
β	thermal expansion coefficient
χ	volume magnetic susceptibility
χ_g	mass magnetic susceptibility, χ/ρ
μ	magnetic permeability
μ_0	permeability of vacuum
v	kinematic viscosity
θ	dimensionless temperature
ρ	density
τ	dimensionless time
Subscripts	
c	critical level
x, X	horizontal components
z, Z	vertical components
Non-dimensional parameters	
Nu	Nusselt number, ratio of total convection heat flux to total conduction heat flux
Pr	Prandtl number, v/D
Ra_T	Rayleigh number, $g\beta\|T1 - T2\|h^3/vD$
V_X, V_Z	non-dimensional velocities
X, Z	non-dimensional coordinates

on thermal transport in an aqueous solution of gadolinium nitrate, a paramagnetic salt. In their experiment, a cylindrical cell filled with the solution was placed in a vertical superconducting magnet as shown in Fig 1. The vertical magnetic field gradients exerted a vertical magnetization force ($\mathbf{F}_m$) on this paramagnetic solution as indicated by the arrows. The convection cell can be used with the heater on the top or the bottom, and can be placed either above or below the magnet center.

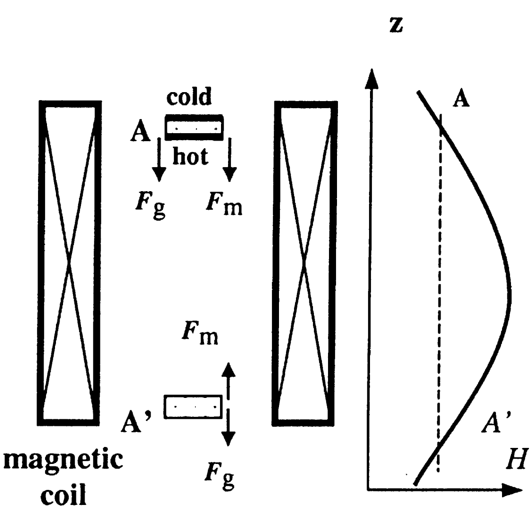

Fig. 1. Schematic of the experimental setup and the distribution of magnetic field strength (H) along the central axis of a superconducting magnet [4].

These experiments reveal the following three phenomena: (a) when such a cell that is heated from below (temperature differences $\Delta T < 0$) is placed below the center of the coil of the magnet at position A' and the magnetization force ($\mathbf{F}_m$) is oriented upward, Nusselt number (Nu) decreases with increasing $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)$ when $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) < 5$ T^2/m, indicating a partial suppression of convection. When $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) = 6$ T^2/m, Nu remains unity for ΔT up to 5°C, indicating that convection is completely suppressed. When $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) = 16$ T^2/m, Nu number remains unity for ΔT up to 30°C; (b) when such a cell heated from below is placed at position A and $\mathbf{F}_m$ acts downward, Nu increases with increasing $|\mu_0^2 \mathrm{H}(\mathrm{d}H/\mathrm{d}z)|$ for given ΔT, indicating an enhancement of buoyancy-driven convection; and (c) when the cell heated from above (temperature differences $\Delta T > 0$) is placed at position A' and $\mathbf{F}_m$ is upward, Nu remains unity for $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) = 5$ T^2/m for $|\Delta T|$ up to 32°C, indicating no convection. However, when $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) = 6$ T^2/m, Nu begins to increase for $|\Delta T| > 2$°C, indicating the onset of magnetothermal convection. These observations indicate that buoyancy-driven convection in fluids can be controlled via an applied inhomogeneous magnetic field.

In order to clarify the mechanism of the above magnetically controlled convection, we numerically simulated the system shown in Fig. 1. Because a magnetization force can act on any material, our present study suggests a new method for controlling convection in either electrically non-conducting or low-conducting fluids. In Section 2, we describe a theoretical formulation. In Section 3, we describe the results of our numerical simulations. In Section 4, we discuss the simulation results compared with previously observed phenomena [4].

2. Theoretical formulation

2.1. Magnetic forces

The magnetic forces acting on a fluid element can be derived from the Helmholtz free energy, which is stored in the flow medium by a magnetic field. According to Landau and Lifshitz [8], the magnetic force, $\mathbf{F}$, can be expressed as [Eq. (34.3) in [8] converted to SI units]:

$$\mathbf{F} = \frac{1}{2}\nabla\left[\mathbf{H}^2\rho\left(\frac{\partial\mu}{\partial\rho}\right)_T\right] - \frac{1}{2}\mathbf{H}^2\nabla\mu + \mu[\mathbf{j}\times\mathbf{H}], \tag{2}$$

where the first term on the right-hand side represents magnetostriction forces caused by magnetic field gradients, the second term represents the dependence of the magnetic force on the gradients of μ, and the last term represents the Lorentz force, $\mathbf{F}_\mathrm{L}$, in electrically conducting fluids. For electrically non-conducting fluids, $\mathbf{j} = 0$ and the last term vanishes.

The magnetic permeability, μ, can be expressed as follows:

$$\mu_\mathrm{r} - 1 = \chi_\mathrm{g}(T)\rho = \chi, \quad \mu_\mathrm{r} = \mu/\mu_0 \tag{3}$$

where $\chi_\mathrm{g} > 0$ for paramagnetic materials, such as oxygen gas and paramagnetic salts, χ_g can be expressed by Curie's law, $\chi_\mathrm{g} = C/T$, where C is a constant that is characteristic of the substance. Most substances (e.g., water) are diamagnetic ($\chi_\mathrm{g} < 0$), and χ_g is independent of temperature.

Substituting Eq. (3) into Eq. (2), the first term on the right-hand side of Eq. (2) becomes:

$$\frac{1}{2}\nabla\left[\mathbf{H}^2\rho\left(\frac{\partial\mu}{\partial\rho}\right)_T\right] = \frac{1}{2}\nabla(\mu_0\rho\chi_\mathrm{g}\mathbf{H}^2) = \frac{1}{2}\mu_0\rho\chi_\mathrm{g}\nabla\mathbf{H}^2 + \frac{1}{2}\mu_0\mathbf{H}^2\nabla(\rho\chi_\mathrm{g}),$$

and the second term on the right-hand side of Eq. (2) becomes

$$-\frac{1}{2}\mathbf{H}^2\nabla\mu = -\frac{1}{2}\mu_0\mathbf{H}^2\nabla\mu_\mathrm{r} = -\frac{1}{2}\mu_0\mathbf{H}^2\nabla(\rho\chi_\mathrm{g}).$$

Then, Eq. (2) can be rewritten as:

$$\mathbf{F} = \frac{1}{2}\mu_0\chi\nabla\mathbf{H}^2 + \mu[\mathbf{j}\times\mathbf{H}]. \tag{4}$$

The first term on the left-hand side of Eq. (4) is the magnetization force $\mathbf{F}_\mathrm{m}$ which originates from the interaction between the magnetic field $\mathbf{H}$ and the magnetic moments characterized by the magnetization ($=\chi\mathbf{H}$).

A density gradient ($\Delta\rho$) will produce a gravitational buoyancy force $\Delta\mathbf{F}_\mathrm{g} = \Delta\rho\mathbf{g}$. Similarly, a susceptibility gradient ($\Delta\chi$) existing in a fluid can produce a magnetic buoyancy force in a non-uniform magnetic field:

$$\Delta\mathbf{F}_\mathrm{m} = \frac{1}{2}\mu_0\Delta\chi\nabla\mathbf{H}^2 = \frac{1}{2}\mu_0(\chi_\mathrm{g}\Delta\rho + \rho\Delta\chi_\mathrm{g})\nabla\mathbf{H}^2. \tag{5}$$

Due to the gradient of $\mathbf{H}^2$ and that of χ which is induced by an applied thermal gradient, $\mathbf{F}_\mathrm{m}$ becomes a non-conservative force ($\nabla\times\mathbf{F}_\mathrm{m} \neq 0$), and induces magnetothermal convection. An imposed thermal gradient produces spatial gradients of χ, $\Delta\chi$, in the magnetization both through the temperature-dependent magnetic susceptibility, $\Delta\chi_\mathrm{g}$, and through the temperature-dependent mass density, $\Delta\rho$, and therefore produces the magnetic buoyancy force $\Delta\mathbf{F}_\mathrm{m}$. Note that because $\Delta\chi_\mathrm{g} = 0$ for diamagnetic fluids, only $\Delta\rho$ influences $\Delta\chi$. In non-conducting fluids, this magnetic buoyancy force can either promote or inhibit convection in a manner similar to buoyancy forces caused by gravity.

2.2. Mathematical model

In the experiment described in Section 1, a closed, 10-mm high, 25-mm diameter cylindrical cell filled with an aqueous solution of gadolinium nitrate was placed inside a vertical superconducting magnet (Fig. 1). We numerically simulated thermal convection in a two-dimensional rectangular cell filled with an aqueous solution (Fig. 2). The fluid was heated from either below or above (Fig. 2(a)) by keeping the temperature on opposite horizontal walls at different constant temperatures, $T1$ and $T2$, respectively. The other walls were thermally insulated. A no-slip condition was imposed at all four walls of the cell. Fig. 2(b) showed the condition where the cell was heated from the side.

For an electrically non-conducting or low-conducting fluid in the presence of an external inhomogeneous magnetic field, the basic equations solved in our simulation are the Navier–Stokes equations for fluid flow, including the magnetic force $\mathbf{F}$ (magnetization force $\mathbf{F}_\mathrm{m}$ and Lorentz force $\mathbf{F}_\mathrm{L}$) given by Eq. (4) and the gravity force $\mathbf{F}_\mathrm{g} = \rho\mathbf{g}$. The complete set of equations are

Equation of continuity:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0. \tag{6}$$

Equation of motion:

$$\rho\frac{\partial\mathbf{v}}{\partial t} + \rho(\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla p + \rho\nu\nabla^2\mathbf{v} + \mathbf{F}_\mathrm{g} + \mathbf{F}_\mathrm{m} + \mathbf{F}_\mathrm{L}. \tag{7}$$

Equation of energy conservation:

$$\frac{\partial T}{\partial t} + (\mathbf{v}\cdot\nabla)T = D\nabla^2 T. \tag{8}$$

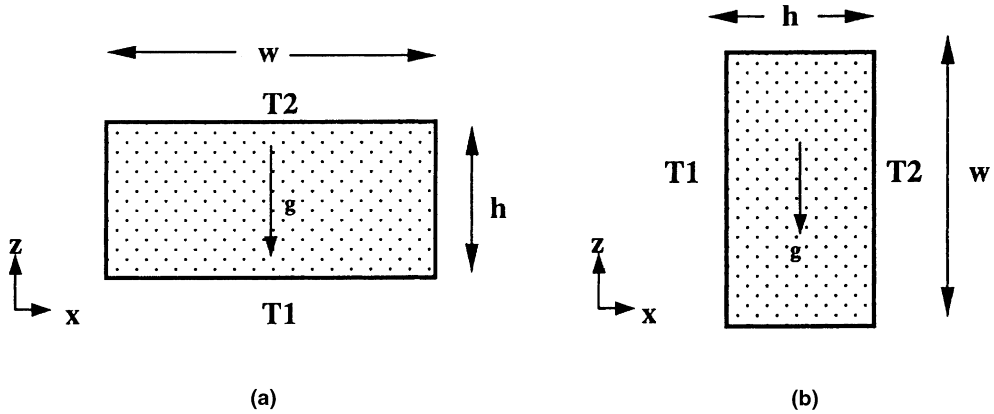

Fig. 2. Schematic of the physical system (a) heated from either below or above, and (b) heated from the side.

According to Ohm's Law, $\mathbf{j} = \sigma(\mathbf{E} + \mu\mathbf{v} \times \mathbf{H})$, where σ is the electrical conductivity and $\mathbf{E}$ is the electric field vector. From Maxwell's equations, $\mathbf{E} = -\nabla\phi - \partial\mathbf{A}/\partial t$, where ϕ and $\mathbf{A}$ are the electric scalar potential and the electric vector potential, respectively. In the static magnetic field $\mathbf{H}$, $\partial\mathbf{A}/\partial t = 0$. Therefore, $\mathbf{E} = -\nabla\phi$. To satisfy the continuity of electric current density, $\nabla \cdot \mathbf{j} = 0$.

Therefore, the electric potential can be expressed as:

$$\nabla^2\phi = \nabla \bullet (\mu\mathbf{v} \times \mathbf{H}). \tag{9}$$

Eqs. (6)–(9) are coupled because fluid flow affects convective heat transfer, thereby affecting the magnetic and gravitational forces, whereas fluid flow itself is driven by the combined action of these forces. To analyze thermal convection under an inhomogeneous magnetic field, the following assumptions were made:

1. Incompressible, laminar flow.
2. Constant physical properties, except density.
3. The magnetic susceptibility for paramagnetic fluids can be expressed as $\chi_g/\chi_r = T_r/T = 1 - (\Delta T/T_r) + O\{(\Delta T/T_r)^2\}$, where χ_r is the mass magnetic susceptibility at $T = T_r$. T_r is the reference temperature, taken as the smaller value among $T1$ and $T2$.
4. Due to the small magnetic Prandtl number, $\mu\sigma\nu \ll 1$, the induced magnetic field is negligible.
5. Viscous dissipation heating and Joule heating are negligible for aqueous solutions.
6. The vertical field–field gradient product, $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)$, is spatially uniform within the cell, and the horizontal magnetic field gradient is negligible. Because the size of a cell is much smaller than that of the magnet bore in the experiment [4], this assumption is reasonable. The applied vertical field can then be estimated as follows:

$$\mu_0 H = \sqrt{C_H \bullet z + \mu_0^2 H_0^2}, \tag{10}$$

where $C_H = 2\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)$ and H_0 is the magnetic field intensity at the center of the bottom wall ($z = 0$).

We used h, h^2/D, D/h, and $\rho_r D^2/h^2$ as characteristic scales for length, time, velocity, and pressure, respectively. The non-dimensional temperature is defined as $\theta = \Delta T/|T1 - T2|$. Accordingly, the dimensionless governing equations are:

$$\frac{\partial V_X}{\partial X} + \frac{\partial V_Z}{\partial Z} = 0 \tag{11}$$

$$\frac{\partial V_X}{\partial \tau} + V_X\frac{\partial V_X}{\partial X} + V_Z\frac{\partial V_X}{\partial Z} = -\frac{\partial P}{\partial X} + Pr\left(\frac{\partial^2 \mathrm{V}_X}{\partial X^2} + \frac{\partial^2 V_X}{\partial Z^2}\right) - Pr \cdot R_{\mathrm{m}X}\theta - Ha^2 Pr V_X, \tag{12}$$

$$\frac{\partial V_Z}{\partial \tau} + V_X\frac{\partial V_Z}{\partial X} + V_Z\frac{\partial V_Z}{\partial Z} = -\frac{\partial P}{\partial Z} + Pr\left(\frac{\partial^2 V_Z}{\partial X^2} + \frac{\partial^2 V_Z}{\partial Z^2}\right) + Pr \cdot Ra_T\theta - Pr \cdot R_{\mathrm{m}Z}\theta, \tag{13}$$

$$\frac{\partial \theta}{\partial \tau} + V_X\frac{\partial \theta}{\partial X} + V_Z\frac{\partial \theta}{\partial Z} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Z^2}, \tag{14}$$

where $R_{\mathrm{m}X} = C_{\mathrm{m}}H(\partial H/\partial x)|T1 - T2|h^3/\nu D$, $R_{\mathrm{m}Z} = C_{\mathrm{m}}H(\partial H/\partial z)|T1 - T2|h^3/\nu D$, $Pr = \nu/D$, $Ra_T = g\beta\,|T1 - T2|\,h^3/\nu D$, $Ha = \mu_0 Hh(\sigma/\rho_r\nu)^{1/2}$ and $C_{\mathrm{m}} = \mu_0\chi_r(\beta + 1/T_r)$. We solved the system of Eqs. (11)–(14) by using the boundary conditions shown in Fig. 2 and by using a finite volume method combined with the SIMPLEC algorithm [9]. The solution of the system of equations was iterated until the maximum relative deviations of both the velocity and temperature were less than 10^{-5}. To check the computational accuracy, we used three uniform grids of 81×33, 101×41, and 151×61 to simulate the convection of heat from below. The differences in the relative values of the mean Nusselt numbers and the maximum velocities were within 1% for all three simulations. The results on a grid of 101×41 differed by less than 0.5% of the results on a grid of 151×61. Therefore, we used the grid of 101×41 in our present study.

We used the same parameters for the simulations as for the experiments by Braithwaite et al. [4], and χ_g is 1.63×10^{-7} m^3/kg. Due to the lack of published values for the gadolinium nitrate solution, we used values for water to approximate these parameters, $\rho = 1000$ kg/m^3, $\nu = 10^{-6}$ m^2/s, $\beta = 2.5 \times 10^{-4}$/K, and $D = 1.454 \times 10^{-7}$ m^2/s.

3. Simulation results

3.1. Effect of Lorentz force on thermal convection (domain as Fig. 2(a) and $T1 > T2$)

Fig. 3(a) shows the velocity distribution without an applied external magnetic field and when the cell is heated from below ($T1 - T2 = 2$°C, $T1 = 22$°C and $Ra_T = 33,734$). Due to gravity, natural convection is generated in the cell. When the cell was placed at position A' in Fig. 1, as indicated by Eq. (12), the Lorentz force also suppressed convection in electroconducting fluids. Due to the lack of published data for electrical conductivity of gadolinium nitrate solution, we used value for an aqueous solution of NaCl (30 wt%) to approximate it. Its electrical conductivity of 21 Ω^{-1} m^{-1} [10] is much smaller than that of a silicon melt of 1.29×10^6 Ω^{-1} m^{-1} [11], in which the Lorentz force is applied to quench convection.

For comparison, we numerically simulated thermal convection for the lower plate heated and for (a) only the Lorenz force active, (b) both Lorenz and upward magnetization forces active, and (c) only the upward magnetization force active. Table 1 shows numerical results when the applied field is $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) = 4$ T^2/m with $\mu_0 H_0 = 1$ T. The damping effect on natural convection by the Lorentz force is negligible because the change of Nusselt number is very small. However, the upward magnetization force can reduce Nusselt number by about 30%. When both kinds of forces exert, the resulting Nusselt number is nearly the same as that of the magnetization force. When $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) = 5$ T^2/m, convection is completely quenched by the upward magnetization force. Therefore, the damping effect on convection was mainly due to the magnetization forces.

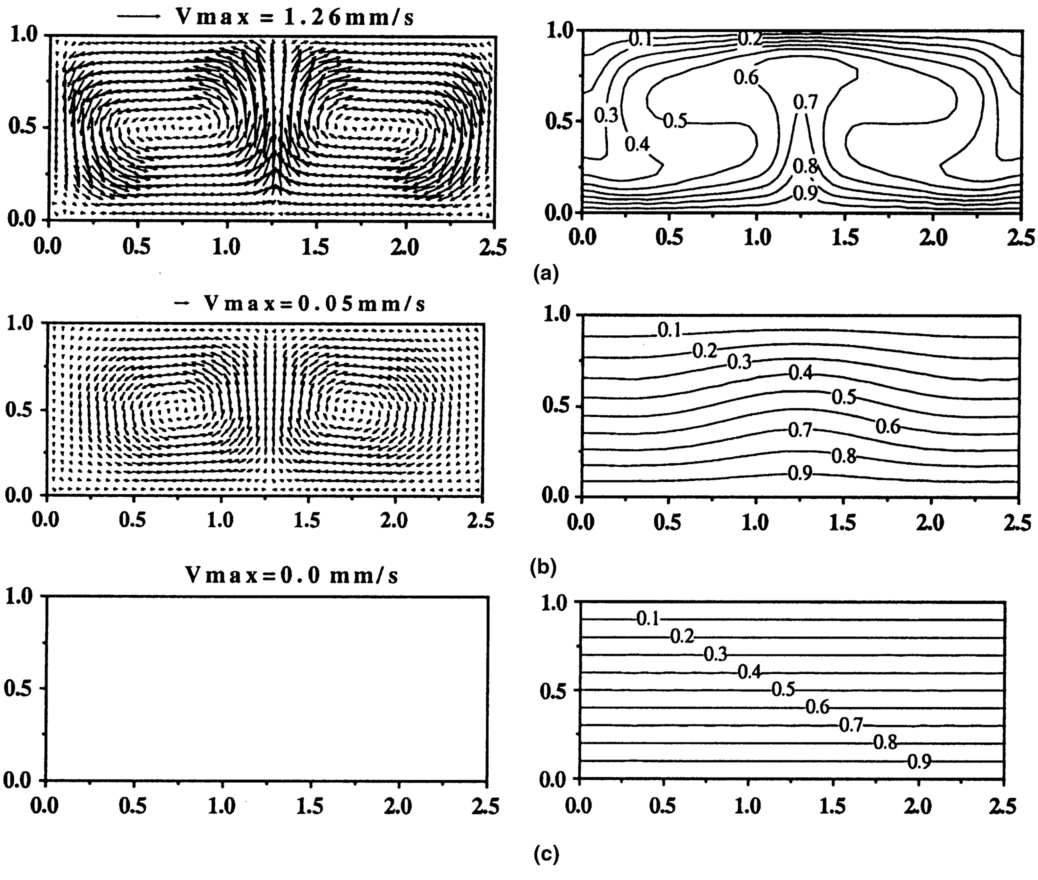

Fig. 3. Velocity vectors (left) and temperature contours (right) in a paramagnetic fluid heated from below when $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)$ is (a) 0, (b) 4.86, and (c) 5.5 T^2/m.

Table 1
Effects of Lorentz force ($\mathbf{F}_L$) and magnetization force ($\mathbf{F}_m$) on convection ($\Delta T = 2°C$)

	Without field	$\mathbf{F}_L$	$\mathbf{F}_L + \mathbf{F}_m$	$\mathbf{F}_m$	$\mathbf{F}_m$	$\mathbf{F}_L$
$\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)$ (T^2/m)		4	4	4	5	5
$\mu_0 H_0$ (T)	0	1.0	1.0	1.0	1.0	10
V_{max} (mm/s)	1.26	1.24	0.37	0.38	0	0.56
Nu	3.20	3.18	2.18	2.20	1.0	2.59

The Lorenz force term in Eq. (12) is proportional to H^2. Even when $\mu_0 H_0 = 10$ T, the Lorenz force reduces Nu by about 20%. In the experimental observations [4], $\mu_0 H_0 = 1$ T. Hereafter, we only consider the effects of magnetization forces in the numerical simulations.

According to the theoretical analyses [12], the quenching of convection in a layer of electroconducting fluid heated from below, under an external magnetic field, depends on H and σ of the fluid, and its effect can be expressed with the dimensionless parameter $Q = Ha^2 = \mu^2 \sigma H^2 h^2/\rho\nu$. In order to completely quench convection only by the Lorentz force in our simulations ($Ra = 33,735$), the critical value of Q is about 2500, which requires $\mu_0 H$ about 34.5 T. Actually, the value of Q is about 2.1 for $\mu_0 H = 1$ T in the experiment [4], which is much smaller than the critical value. Therefore, low-conducting fluids, i.e., the aqueous solution (21 Ω^{-1} m^{-1}), can be approximated as a non-conducting fluid under the present simulation conditions.

3.2. Case 1: paramagnetic fluid heated from below (domain as Fig. 2(a) and T1 > T2)

Fig. 3(b) shows that natural convection is partially suppressed when the cell is placed at position A' in Fig. 1 and an upward magnetization force $\mathbf{F}_m$ is applied ($\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) = 4.86$ T^2/m). When $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) = 5.5$ T^2/m, convection has been completely suppressed (Fig. 3(c)). Fig. 4 shows that both the Nusselt number (□) and the maximum velocity (○) decrease with increasing gradient of $\mathbf{H}^2$ until convection is completely suppressed for $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) > 4.9$ T^2/m. Hereafter, we consider that this point is critical for the upward magnetization force to completely quench convection.

In contrast, when the cell is placed at position A and the magnetization force is applied downward (i.e., $H(\mathrm{d}H/\mathrm{d}z) < 0$), convection is enhanced. Both the Nusselt number and the maximum velocity increase with increasing $|\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)|$. These numerical results are consistent with the experimental observations of (a) and (b) described in Section 1 [4].

In our simulation, we assumed $\Delta T = 2°C$, which corresponds to $Ra_T/Ra_c \approx 20$, where $Ra_c = 1720$ is the critical Rayleigh number in the absence of magnetic fields. In this section, we discuss the dependence of Ra_T on the critical value of $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)$.

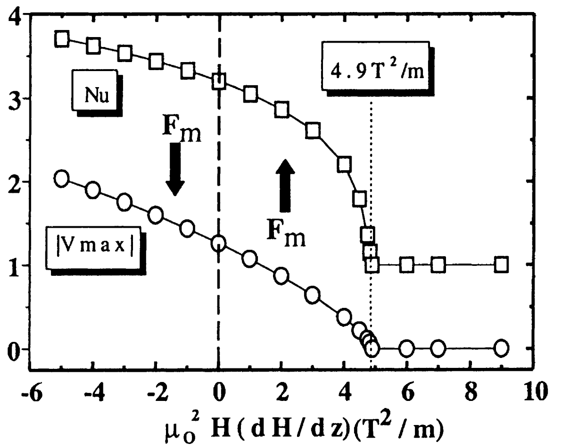

Fig. 4. Dependence of the vertical field–field gradient product $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)$ on the average Nusselt number (□) and on the maximum velocity ($|V_{max}|$: mm/s) (○) in a paramagnetic fluid (heated from below).

As shown in Eq. (13), for a uniform vertical $\mathbf{H}^2$ gradient, the effects of the driving force on convection can be divided into two components: the vertical component of the magnetic force ($S_{mZ} = -PrR_{mZ}\theta$), and the gravitational component ($S_g = PrRa_T\theta$). A negative R_{mZ} will enhance the effect of gravity, and a positive R_{mZ} will counteract the effect of gravity. When $|Ra_T - R_{mZ}| \leqslant Ra_c$ in Eq. (13), thermal convection can be completely quenched. Hence, the following relation can determine the critical value, $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$:

$$Ra_T \bullet \left| 1 - \frac{\mu_0 \chi_r H(\mathrm{d}H/\mathrm{d}z)_c}{g}\left(1 + \frac{1}{\beta T_r}\right)\right| = Ra_c. \qquad (15)$$

Then, the critical value of $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$ is:

$$\mu_0^2 H\left(\frac{\mathrm{d}H}{\mathrm{d}z}\right)_c = \left(1 - \frac{Ra_c}{Ra_T}\right)\left[\frac{\mu_0 g}{\chi_r(1 + 1/\beta T_r)}\right]. \qquad (16)$$

Fig. 5(a) shows the dependence of Ra_T/Ra_c on the value of $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$ when the cell is heated from below. In our simulation condition, the $Ra_T = 33,735$. So, we get $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c = 4.9$ T^2/m from Eq. (16). For $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) > \mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$, no convection occurs. When $Ra_T \gg Ra_c$, the expression of $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$ can be simplified as:

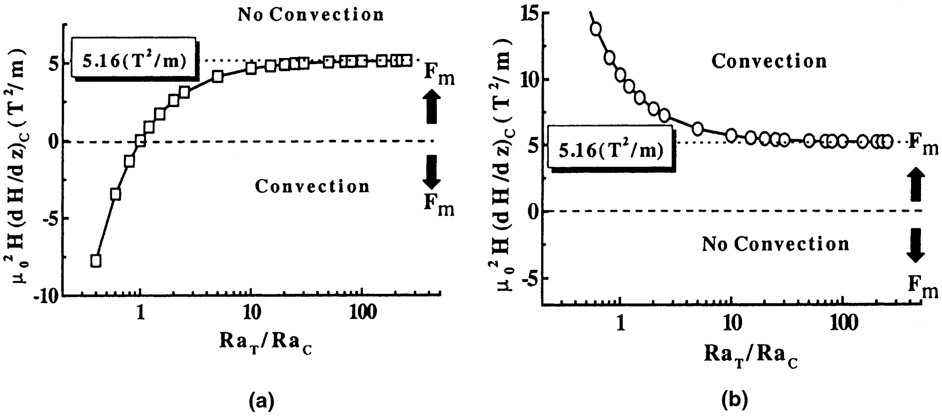

Fig. 5. Dependence of the critical value of $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)$ on Ra_T/Ra_c for a paramagnetic fluid heated (a) from below and (b) from above.

$$\mu_0^2 H\left(\frac{\mathrm{d}H}{\mathrm{d}z}\right)_c \approx \frac{\mu_0 g}{\chi_r(1+1/\beta T_r)}. \tag{17}$$

It is characteristic that the extreme value of $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$ is the function of χ_r, β and T_r. This value was calculated to be 5.16 T^2/m in the experimental conditions [4].

When Ra_T/Ra_c increases, $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$ increases from 0 to 5.16 T^2/m. This shows that a small upward magnetization force can quench convection. In contrast, when natural convection does not occur in the absence of a magnetic field (i.e., $Ra_T/Ra_c < 1$), a downward magnetization force induces magnetothermal convection. These tendencies also agree with the experimental observations of (a) and (b) in Section 1 [4].

3.3. Case 2: paramagnetic fluid heated from above (domain as Fig. 2(a) and $T1 < T2$)

In the absence of a non-uniform magnetic field, there is no convection when the fluid layer is heated from above, because gravity tends to stabilize the fluid layer. When the upward magnetization force is small, i.e., $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) = 5.0$ T^2/m (Fig. 6(a)), there is no convection. However, when $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) = 5.5$ T^2/m, the onset of magnetothermal convection occurs as shown in Fig. 6(b). Fig. 7 shows that the upward magnetization force induces and promotes magnetothermal convection for $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) > 5.4$ T^2/m. On the other hand, the downward magnetization force plays the same role as gravity and enhances the stability of the fluid layer, and

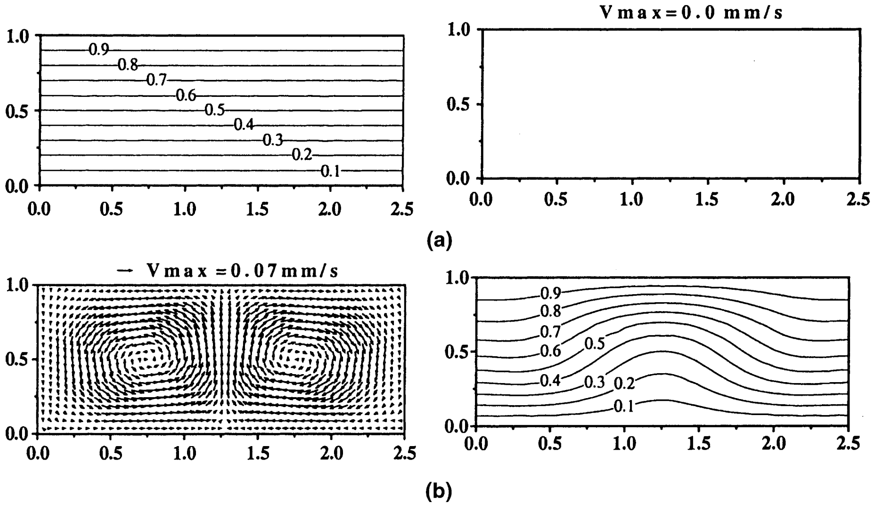

Fig. 6. Velocity vectors (left) and temperature contours (right) when a paramagnetic fluid is heated from above and when $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)$ is (a) 5.0, and (b) 5.5 T^2/m.

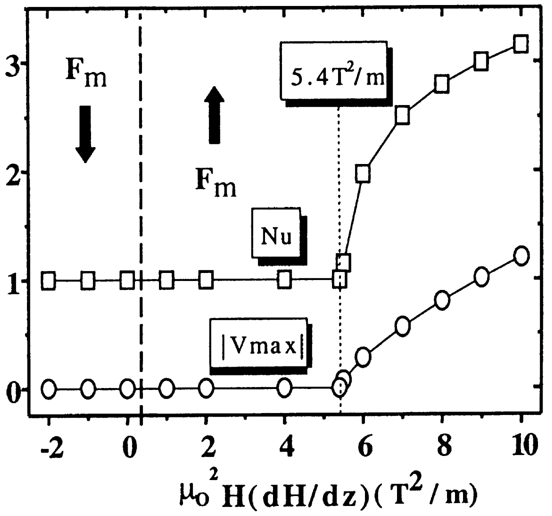

Fig. 7. Dependence of $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)$ on the average Nusselt number (□) and on the maximum velocity ($|V_{\max}|$: mm/s) (○) in a paramagnetic fluid (heated from above).

thus no convection occurs when the simulation domain is put on position A in Fig. 1.

Fig. 5(b) shows the dependence of Ra_T/Ra_c on the critical value of $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$. Contrary to the behavior observed for Case 1, when $Ra_T/Ra_c < 20$, $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$ decreases with increasing Ra_T/Ra_c. For $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) < \mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$, convection will not occur. Because there is a threshold for the onset of Rayleigh–Benard convection in the absence of magnetic fields, our simulations indicate that for $\Delta T = 2°\mathrm{C}$ ($Ra_T/Ra_c \approx 20$) the critical value of $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$ for inducing convection in Case 2 is about 5.4 T^2/m, which is greater than the value of 4.9 T^2/m required for quenching convection in Case 1. As shown in Fig. 5, this difference rapidly decreases with increasing Ra_T/Ra_c, and the extreme values of $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$ for both cases take the same value of 5.16 T^2/m.

3.4. Case 3: paramagnetic fluid heated from the side (domain as Fig. 2(b) and $T1 > T2$)

Fig. 8(a) shows that in the absence of magnetic fields, natural convection is generated clockwise in the cavity. Fig. 8(b) shows the damping effect of the upward force on natural convection when $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) = 5.0$ T^2/m. Fig. 9 shows that when the magnetization force is upward and $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) < 5.16$ T^2/m, convection decreases with increasing $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)$. When $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) \approx 5.16$ T^2/m, natural convection is completely quenched. On the other hand, when $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) > 5.16$ T^2/m, natural convection caused by gravity is suppressed and replaced by the counter convection dominated by the magnetic force as shown in Fig. 8(c).

As shown in Fig. 9, similar to Case 1, the downward magnetization force promotes natural convection. Moreover, the critical value of $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$ obtained in Case 3 has the same value, 5.16 T^2/m, as the extreme values of $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)$ for Cases 1 and 2 ($Ra_T \gg Ra_c$) as shown in Fig. 5.

4. Discussion

Our numerical results show that convection in electrically non-conducting paramagnetic fluids can be controlled by varying the magnitude of the vertical magnetization force and its direction. Moreover, this mechanism is also applicable in paramagnetic fluids with low electrical conductivity, where the effect of the Lorentz force is small compared with that of the magnetization force. We will discuss our results and compare the simulated results with the observed phenomena [4].

In our numerical simulation, due to the lack of published values for the gadolinium nitrate solution, we used values for water to approximate these parameters, ρ, ν, β and D. Eq. (15) is a general function to determine the threshold value. It is characteristic that $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$ is the function of χ_r, β and T. If the value of β differs 10% from that of water, $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$ will vary by about 10%.

In our simulations for $\Delta T = 2°$ C ($Ra_T/Ra_c \approx 20$), for $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) = 4.9$ T^2/m, convection is completely counteracted for the case of heated from below (Case 1). This is consistent with measured values of about 5 T^2/m for Case (a) stated in Section 1. Furthermore, the theoretical analysis on the thermoconductive instability is found in [13]. Using their results, we checked the critical value $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$, of case 1 when $Ra_T = 33,735$, and obtained nearly the same value.

Our simulation suggests that even values of $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)$ as small as 5 T^2/m are sufficient to counteract the effect of gravity on thermal convection in solutions of gadolinium nitrate (Figs. 4, 7 and 9). On the other hand, according to the magnetization force shown in Eq. (1), $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) \approx 75.6$ T^2/m is required to completely counteract gravity ($\mathbf{F}_g = \rho\mathbf{g}$) in the solutions of gadolinium nitrate at $T = 20°\mathrm{C}$. This large difference results from the magnetic buoyancy force, rather than from the magnetization force required to halt thermal convection. As indicated by Eq. (5), the magnetic buoyancy force for paramagnetic fluids is affected by both $\Delta\rho$ and $\Delta\chi_g$. Because $\Delta\chi_g/\chi_g \propto \Delta T/T$ and $\Delta\rho/\rho \propto \beta\Delta T$, the contribution of the magnetic buoyancy force caused by the mass magnetic susceptibility difference is larger than that caused by the density difference. For the paramagnetic solution used in our study, the buoyancy force caused by $\Delta\chi_g$ is more than 10 times larger than the buoyancy force caused by $\Delta\rho$.

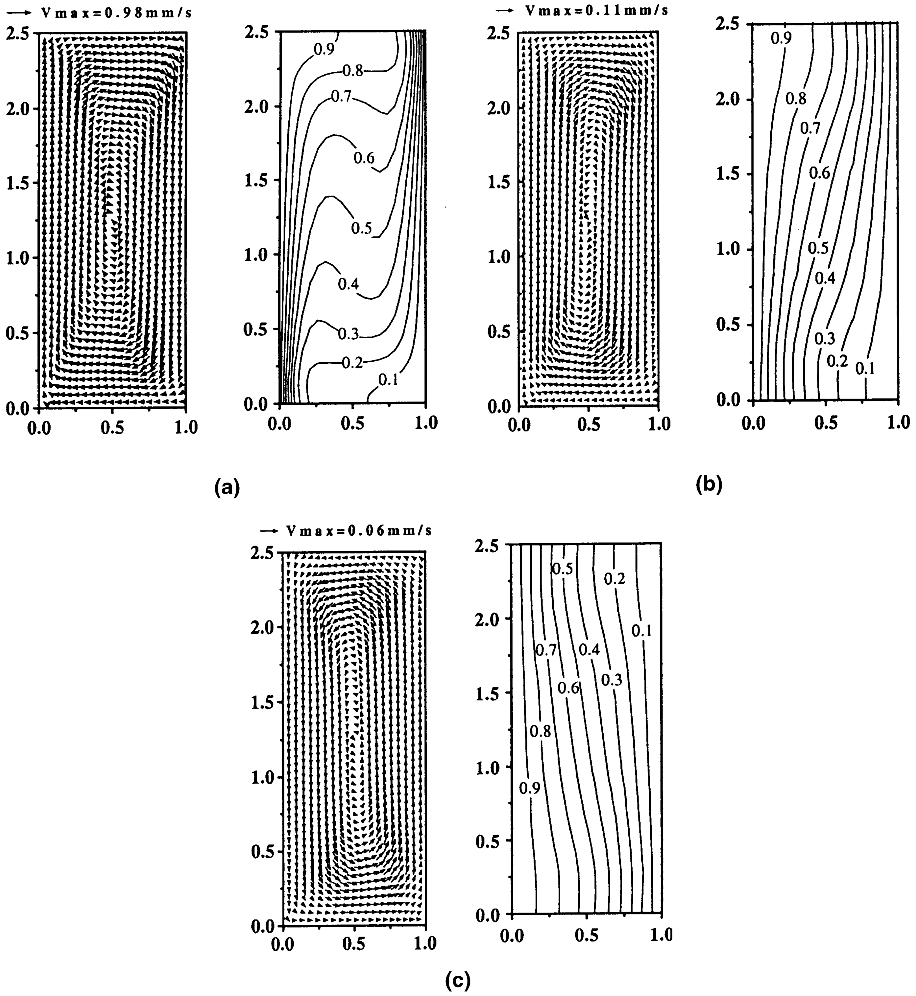

Fig. 8. Velocity vectors (left) and temperature contours (right) when a paramagnetic fluid is heated from the side and when $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)$ is (a) 0, (b) 5.0, and (c) 5.25 T^2/m.

Therefore, even a moderate value of $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)$ of 5 T^2/m can effectively counteract thermal convection in paramagnetic fluids. On the other hand, in the absence of temperature gradients [5], only $\Delta\rho$ contributes to magnetic buoyancy. Therefore, $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) \approx 75.6$ T^2/m is required to compensate the effect of gravity under isothermal conditions.

Thus, vertical magnetization forces can be useful for controlling thermal convection in paramagnetic fluids. The small value of $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)$ obtainable even with permanent magnets is feasible for achieving magnetic buoyancy forces required to either promote or inhibit convection in paramagnetic fluids. A magnetization force produced by an inhomogeneous magnetic field can act not only on paramagnetic but also diamagnetic materials. The present study suggests a new method for controlling convection in electrically non-conducting and low-conducting fluids, and has significant potential for a variety of applications, for example, control of heat transfer and crystal formation processes.

5. Conclusions

Based on our simulation and analysis, we conclude the following:

1. Magnetization forces can induce a magnetic buoyancy force analogous to that of gravity, and have significant potential for applications in controlling convection and in heat transfer engineering.
2. A vertical magnetic buoyancy force can suppress or promote natural convection, or even reverse the direction of thermal convection in either electrically non-conducting fluids or low-conducting fluids, such

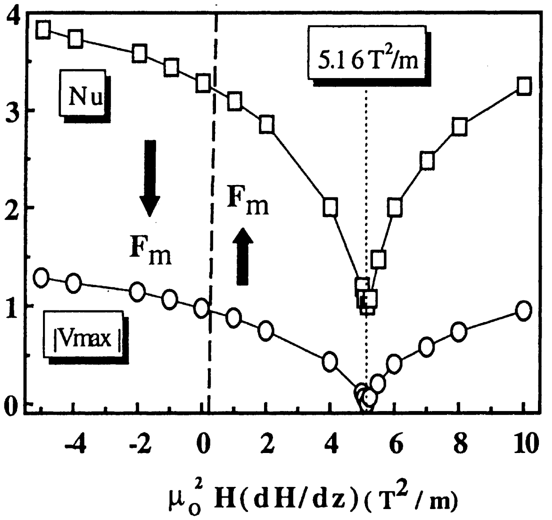

Fig. 9. Dependence of the vertical field–field gradient product $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)$ on the average Nusselt number (□) and maximum velocity ($|V_{max}|$: mm/s) (○) in a paramagnetic fluid (heated from the side).

as melts of inorganic salts, glasses and aqueous solutions.

3. When paramagnetic fluids are heated from below and upward magnetization forces act on fluids, thermal convection is suppressed. When $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) > \mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$, convection is completely quenched. On the other hand, when downward magnetic forces act on fluids, convection is enhanced.
4. For paramagnetic fluids heated from above, upward magnetization forces can induce convection when $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) > \mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$, whereas downward magnetic forces enhance the stability of the fluid and no convection occurs.
5. When convection is completely quenched in the paramagnetic fluids heated from either below or above, the following relation between Ra_T and $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$ exists:

$$Ra_T \bullet \left| 1 - \frac{\mu_0 \chi_r H(\mathrm{d}H/\mathrm{d}z)_c}{g} \left(1 + \frac{1}{\beta T_r} \right) \right| \leqslant Ra_c,$$

when $Ra_T \gg Ra_c$, $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$ takes a constant value of

$$\mu_0^2 H \left(\frac{\mathrm{d}H}{\mathrm{d}z} \right)_c \approx \frac{\mu_0 g}{\chi_r (1 + 1/\beta T_r)}.$$

6. When the fluid is heated from the side, the upward magnetization force partially dampens natural convection for $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) < \mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$, whereas it induces convection dominated by the magnetization force when $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z) > \mu_0^2 H(\mathrm{d}H/\mathrm{d}z)_c$.
7. Only a small value of $\mu_0^2 H(\mathrm{d}H/\mathrm{d}z)$, which is achievable with permanent magnets or electromagnets, is needed to offset the effect of gravity on thermal convection.

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